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Magneto-oscillations of the tunnelling current between two-dimensional electron systems

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Abstract. The tunnelling current caused by electron transfer between two-dimensional layers is calculated as a function of the magnetic field applied perpendicular to the layers and level splitting of the tunnel-coupled states. The elastic scattering of the electrons is taken into consideration. Analytical results describing both the tunnelling relaxation rate of photoexcited electrons and the tunnelling current between the independently contacted quantum wells are obtained for two regimes: (i) that of small magnetic fields, for which the Landau quantization is suppressed by the scattering and the oscillating part of the current shows nearly harmonic behaviour; and (ii) that of high magnetic fields, for which the Landau levels are well defined and the current shows a series of sharp peaks corresponding to resonant magnetotunnelling. In the latter case, we used two alternative approaches: the self-consistent Born approximation method and the path-integral method, and demonstrated that the results obtained show reasonable agreement for both of these methods. The influence of the interlayer correlation of the scattering on the first magnetotunnelling peak is also discussed.

1. Introduction

Tunnelling of the electrons between barrier-separated two-dimensional (2D) electron layers is currently under examination in double-quantum-well systems (DQWs). Both direct measurements of the tunnelling current in DQWs with separate contacts [1] and time-resolved spectroscopical measurements of the tunnelling relaxation rate [2] in photoexcited DQWs are carried out. Application of the magnetic field perpendicular to the layers changes the energy spectrum of the electrons and thereby modifies the tunnelling phenomena in DQWs. The electron spectrum in this case is described by two sets of Landau levels; these sets originate from the size-quantization subbands of the left-hand and right-hand wells (l- and r-wells). According to the energy-conservation requirements, the probability of the tunnelling must oscillate with the magnetic field and subband splitting energy Δ (the latter is usually controlled by a transverse bias applied to the side gates); see figure 1 for an explanation. The properties of these oscillations are determined by scattering processes involved in the tunnelling events. For example, the optical phonon-assisted tunnelling leads to giant magnetophonon oscillations [3] of the tunnelling rate. The positions of the magnetophonon peaks are determined by the condition $\Delta - (n' - n)\hbar\omega_c = \hbar\omega_0$ (here ω_0 is the longitudinal optical phonon frequency, ω_c is the cyclotron frequency, and n, n' are the numbers of Landau levels participating in the transition). The first magnetophonon peak of the tunnelling current has been observed in the separately contacted DQWs [4]

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and resonant tunnelling diodes [5, 6], as well as in the multiple-quantum-well system [7]. However, when the electron temperature, level splitting, and difference between the Fermi levels in the wells are smaller than the optical phonon energy, the optical phonon-assisted tunnelling is not important. Instead, an elastic scattering-assisted tunnelling takes place. In high enough magnetic fields, for which the Landau levels are well defined, this tunnelling has maximum probability for the resonant magnetotunnelling conditions

$$\Delta = \hbar\omega_c(n - n') \tag{1}$$

(see figure 1), and the tunnelling efficiency depends on the Landau level broadening due to elastic scattering in the wells. Several resonant magnetotunnelling peaks corresponding to conditions (1) have been experimentally observed in high magnetic fields; see references [4–6]. In weak magnetic fields, nearly harmonic oscillations of the tunnelling current in separately contacted DQWs have been observed; see figure 3 from reference [1]. However, as far as we know, a detailed theoretical analysis of these phenomena has not been presented.

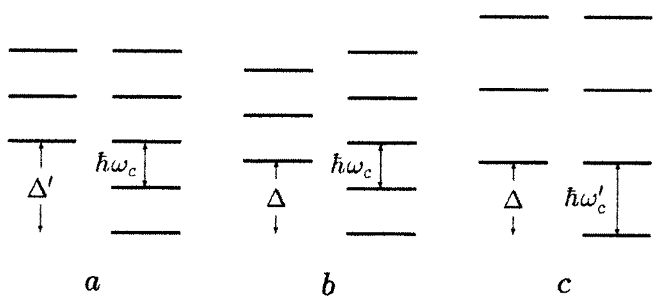

Figure 1. A transition from the non-resonant (*b*) to the resonant tunnelling conditions caused by an increase of the splitting energy Δ (*a*) or cyclotron energy $\hbar\omega_c$ (*c*).

The aim of this paper consists in the theoretical investigation of the tunnelling between the two-dimensional electron systems in the presence of the magnetic field $\boldsymbol{H}$ applied perpendicular to the layers. In our calculations we take into account the elastic scattering of the electrons and restrict ourselves to the one-particle picture of the phenomena under consideration (in the concluding section we describe the conditions where the Coulomb interaction is not essential in the determination of the tunnelling current). We consider weak tunnel coupling, where the value of the tunnelling matrix element T is small in comparison with the broadening of the Landau levels. In this case, each well is described as a separate subsystem with a quasi-equilibrium distribution of the electrons, and the tunnelling current between the wells is calculated in the lowest order of T^2. Two existing schemes of experimental measurements are considered below. The first implies a contactless DQW excited by a short laser pulse. The rate ν of the tunnelling relaxation between the wells is measured by the time-resolved photoluminescence technique (see reference [2], for example). In the second kind of experiment, the DQWs with separate contacts to each well are investigated [1]. This scheme allows one to measure the density j of the tunnelling current directly.

The paper is organized as follows. In section 2 we put general expressions for ν and j through the Green functions of the electrons in the l- and r-layers. In section 3 we develop a quasiclassical calculation of j and ν in the case of weak magnetic fields and derive an analytical expression describing the oscillating behaviour of these values. The calculation of ν and j in strong (quantizing) magnetic fields is carried out in section 4 with the use of two different techniques: the path-integral method and the self-consistent Born approximation method. In section 5 we calculate the shape of the main ($n = n' = 0$) magnetotunnelling peak taking into account interlayer correlation of the scattering. Concluding remarks are made in the last section.

2. General formalism

A method of consideration of the tunnelling between the weakly coupled low-dimensional electron systems under elastic scattering conditions is described in our previous papers [8]. Here 'weak coupling' means that the tunnelling matrix element T (which determines the minimum splitting energy $2T$ of the tunnel-coupled electron subbands) is so small that the probability of the tunnelling is much smaller than the intralayer scattering probability. In these conditions, electron states are properly classified by the layer numbers l and r, while the probability of electron transition between the layers due to the tunnelling is proportional to T^2. Tunnelling relaxation of the photoexcited electrons is determined by the balance equation for the electron concentration n_l in the l-well, which has the usual form $\mathrm{d}n_l/\mathrm{d}t = -\nu n_l$. Here ν means the thermally averaged departure rate of the l-well electrons due to the tunnelling to the empty states of the r-well. It can be expressed through the pair correlator of the causal Green functions $G^j_\varepsilon(\boldsymbol{x}, \boldsymbol{x}')$ ($j = l, r$) in the following way:

$$\nu = \frac{2\pi T^2}{\hbar S} \int \mathrm{d}\varepsilon \;\, f_l(\varepsilon) \int \mathrm{d}\boldsymbol{x} \int \mathrm{d}\boldsymbol{x}' \;\, \left\langle G^l_\varepsilon(\boldsymbol{x}, \boldsymbol{x}') G^r_\varepsilon(\boldsymbol{x}', \boldsymbol{x})\right\rangle \left[\int \mathrm{d}\varepsilon \;\, f_l(\varepsilon) \left\langle G^l_\varepsilon(\boldsymbol{x}, \boldsymbol{x})\right\rangle\right]^{-1} \tag{2}$$

where $f_j(\varepsilon)$ are the quasi-equilibrium distribution functions, $\boldsymbol{x}$ is the in-plane coordinate, S is the normalization area, and $\langle\cdots\rangle$ means statistical averaging over realizations of the random potential of impurities, interface roughnesses, etc. As a rule, the photoexcited electrons are non-degenerate and the distribution function can be taken in the form $f_j(\varepsilon) \sim \exp(-\varepsilon/T_e)$, where T_e is the effective temperature of the electrons.

The density j of the tunnelling current (which is directly measured in the experiments on DQWs with separate contacts) is expressed in a similar way:

$$j = \frac{4\pi e T^2}{\hbar S} \int \mathrm{d}\varepsilon \;\, [f_l(\varepsilon) - f_r(\varepsilon)] \int\int \mathrm{d}\boldsymbol{x} \;\mathrm{d}\boldsymbol{x}' \;\, \left\langle G^l_\varepsilon(\boldsymbol{x}, \boldsymbol{x}') G^r_\varepsilon(\boldsymbol{x}', \boldsymbol{x})\right\rangle \tag{3}$$

where e is the absolute value of the electron charge. In such experiments, the electron gas is degenerate, $f_j(\varepsilon) \simeq \theta(\varepsilon_{Fj} - \varepsilon)$, and the integral over ε should be taken in the interval between the r-well Fermi energy ε_{Fr} and l-well Fermi energy ε_{Fl}.

If we assume that the scattering potentials in the l- and r-layers are uncorrelated (this approximation is quite realistic for a short-range impurity potential or interface roughness potential), the averaging of the pair correlators can be taken independently for each layer, giving

$$\int\int \mathrm{d}\boldsymbol{x} \;\mathrm{d}\boldsymbol{x}' \;\, \left\langle G^l_\varepsilon(\boldsymbol{x}, \boldsymbol{x}') G^r_\varepsilon(\boldsymbol{x}', \boldsymbol{x})\right\rangle = S \int \mathrm{d}\boldsymbol{x} \;\, G^l_\varepsilon(\boldsymbol{x}) G^r_\varepsilon(\boldsymbol{x}) \tag{4}$$

where $G^j_\varepsilon(\boldsymbol{x})$ is the averaged causal Green function of the electron in jth well. This simple case will be considered in the following two sections, while the influence of the interlayer correlation will be discussed in section 5. It is convenient to write $G^j_\varepsilon(\boldsymbol{x})$ in the Landau level representation, introducing Green functions $G^j_\varepsilon(n)$ in this representation:

$$G^j_\varepsilon(\boldsymbol{x}) = \sum_n g_n(|\boldsymbol{x}|) G^j_\varepsilon(n) \tag{5}$$

$$g_n(x) = \frac{1}{2\pi l_H^2} \exp\left(-\frac{x^2}{4l_H^2}\right) L_n^0\left(\frac{x^2}{2l_H^2}\right). \tag{6}$$

Here L_n^0 are the Laguerre polynomials [9] and $l_H = \sqrt{\hbar c/eH}$ is the magnetic length. Using well-known properties of L_n^0, we also obtain

$$\int \mathrm{d}\boldsymbol{x}\ G_\varepsilon^l(\boldsymbol{x}) G_\varepsilon^r(\boldsymbol{x}) = \frac{1}{2\pi l_H^2} \sum_{n=0}^{\infty} G_\varepsilon^l(n) G_\varepsilon^r(n) \tag{7}$$

so the integrals over the in-plane coordinates in equations (2) and (3) reduce to the sums over the Landau level numbers.

In spite of the approximations already made, an analytical calculation of j and ν in the general case is impossible, because it requires an exact knowledge of the Green functions introduced above. To develop an analytical description of the tunnelling, we consider some approximations for the Green functions, which are commonly used in the theory of two-dimensional electrons in the magnetic field.

3. The case of weak magnetic fields

In this section we examine the case where the cyclotron energy is small in comparison with the Fermi energy (or effective temperature) of the electrons:

$$\hbar\omega_c \ll \varepsilon_{Fl}, T_e \tag{8}$$

so a large number of the Landau levels are involved in the consideration. We search for the causal Green functions in the form

$$G_\varepsilon^l(n) = \frac{1}{\pi} \frac{\Gamma_l}{\Gamma_l^2 + (\varepsilon - \varepsilon_n)^2} \tag{9}$$

$$G_\varepsilon^r(n) = \frac{1}{\pi} \frac{\Gamma_r}{\Gamma_r^2 + (\varepsilon + \Delta - \varepsilon_n)^2} \tag{10}$$

where $\varepsilon_n = \hbar\omega_c(n + 1/2)$, and $\Gamma_j = \Gamma_j(\varepsilon)$ are the imaginary parts of the l- and r-well self-energies, which are found self-consistently from the equations (see [10])

$$\Gamma_l = \frac{\hbar}{2\tau_l} \frac{\sinh(2\pi\Gamma_l/\hbar\omega_c)}{\cosh(2\pi\Gamma_l/\hbar\omega_c) + \cos(2\pi\varepsilon/\hbar\omega_c)} \tag{11}$$

$$\Gamma_r = \frac{\hbar}{2\tau_r} \frac{\sinh(2\pi\Gamma_r/\hbar\omega_c)}{\cosh(2\pi\Gamma_r/\hbar\omega_c) + \cos[2\pi(\varepsilon + \Delta)/\hbar\omega_c]}. \tag{12}$$

Here τ_l and τ_r are the quantum lifetimes of the electrons in the wells (for elastic scattering mechanisms) in the absence of the magnetic field [10]. Under the conditions $\exp[-\pi/\omega_c\tau_j] \ll 1$, equations (11) and (12) give the simple relations $\Gamma_j \simeq \hbar/2\tau_j$. However, as the magnetic field increases, Γ_l and Γ_r acquire energy-dependent oscillatory contributions. In equations (9)–(12) we have neglected the dependence of Γ_j on the Landau level number, which is correct in two situations: the first is the case of short-range-correlated scattering (in which this dependence really vanishes), and the second is realized for degenerate electron systems, in which the difference between the Fermi energies ε_{Fl} and ε_{Fr} is small in comparison with these Fermi energies and the electrons tunnel in a narrow interval of energy. In this latter situation we should treat the imaginary part of the self-energy as corresponding to a Landau level near the Fermi energy, and τ_j as the scattering times in the Fermi surface. We have also eliminated the real parts of the self-energies, which is done under conditions (8) via a proper shift of the zero point of energy, and (if $\tau_l \neq \tau_r$) renormalization of the splitting energy Δ.

After substitution of (9) and (10) in equation (7), we can calculate the sum over n, extending the lower limit of the summation to $-\infty$, which is a good approximation under condition (8). In the same way as in the description of magneto-oscillatory phenomena in solids, we use Poisson's rule of summation [9] and obtain the current as follows:

$$j = e\left(\frac{2T}{\hbar}\right)^2 \rho_{2D} \int_{\varepsilon_{Fl}}^{\varepsilon_{Fr}} \mathrm{d}\varepsilon \; \frac{\Delta^2 \Gamma_l \Gamma_r (\tau_l + \tau_r)}{[\Delta^2 + (\Gamma_l + \Gamma_r)^2][\Delta^2 + (\Gamma_l - \Gamma_r)^2]} \left\{ 1 + \frac{\tau_l - \tau_r}{\tau_l + \tau_r} \frac{\Gamma_r^2 - \Gamma_l^2}{\Delta^2} + \frac{2\tau_l \Gamma_l}{(\tau_l + \tau_r)\Delta} \frac{\sin(2\pi\varepsilon/\hbar\omega_c)}{\sinh(2\pi\Gamma_l/\hbar\omega_c)} - \frac{2\tau_r \Gamma_r}{(\tau_l + \tau_r)\Delta} \frac{\sin[2\pi(\varepsilon + \Delta)/\hbar\omega_c]}{\sinh(2\pi\Gamma_r/\hbar\omega_c)} \right\} \tag{13}$$

where $\rho_{2D} = m/\pi\hbar^2$ is the 2D density of states (m is the effective mass of the electron). This equation describes the oscillating current, which depends on the magnetic field H, level splitting Δ, positions of the Fermi levels in the wells, and characteristics of the scattering. It is important to notice that this current is also sensitive to the scattering asymmetry (this means that $\tau_l \neq \tau_r$), while at $H = 0$ this dependence vanishes.

In order to make an analytical calculation of the integral over energy in equation (13), we consider the case where the Dingle factors $\exp[-\pi/\omega_c\tau_j]$ are small. Under this condition, the oscillating part of the current appears as a small additional contribution to the background current. We also assume symmetric scattering, taking $\tau_l = \tau_r = \tau$. The current is given by

$$\begin{aligned} j = j_0 + j_0 \exp\left(-\frac{\pi}{\omega_c\tau}\right) \frac{\hbar\omega_c}{2\pi(\varepsilon_{Fl} - \varepsilon_{Fr})} \Bigg\{ \delta^{-1} \Bigg(&\cos\left[\frac{2\pi(\varepsilon_{Fl} + \Delta)}{\hbar\omega_c}\right] \\ &- \cos\left[\frac{2\pi\varepsilon_{Fl}}{\hbar\omega_c}\right] - \cos\left[\frac{2\pi(\varepsilon_{Fr} + \Delta)}{\hbar\omega_c}\right] + \cos\left[\frac{2\pi\varepsilon_{Fr}}{\hbar\omega_c}\right] \Bigg) \\ &- \frac{2}{1 + \delta^{-2}} \Bigg(\sin\left[\frac{2\pi(\varepsilon_{Fl} + \Delta)}{\hbar\omega_c}\right] - \sin\left[\frac{2\pi(\varepsilon_{Fr} + \Delta)}{\hbar\omega_c}\right] \\ &+ \sin\left[\frac{2\pi\varepsilon_{Fl}}{\hbar\omega_c}\right] - \sin\left[\frac{2\pi\varepsilon_{Fr}}{\hbar\omega_c}\right] \Bigg) \Bigg\} \end{aligned} \tag{14}$$

where j_0 is the current at $H = 0$, which can be expressed through the tunnelling rate ν_0 at $H = 0$, and δ is the dimensionless level-splitting energy:

$$j_0 = e\rho_{2D}\nu_0(\varepsilon_{Fl} - \varepsilon_{Fr}) \qquad \nu_0 = \frac{2T^2\tau}{\hbar^2} \frac{1}{1 + \delta^2} \qquad \delta = \frac{\Delta\tau}{\hbar}. \tag{15}$$

Note that ν_0 and j_0 show a resonance at $\Delta = 0$ corresponding to the resonant tunnelling. An expression similar to equation (14) can be derived for the tunnelling rate ν in the non-degenerate case. Neglecting higher-order contributions to $\hbar\omega_c/T_e$, we obtain

$$\nu = \nu_0 + \nu_0 \exp\left(-\frac{\pi}{\omega_c\tau}\right) \frac{\hbar\omega_c}{2\pi T_e} \left\{ \delta^{-1} \left(1 - \cos\left[\frac{2\pi\Delta}{\hbar\omega_c}\right] \right) + \frac{2}{1 + \delta^{-2}} \sin\left[\frac{2\pi\Delta}{\hbar\omega_c}\right] \right\}. \tag{16}$$

This rate shows oscillations with Δ and $\hbar\omega_c$. The phase of these oscillations (in a similar way to that for equation (14)) depends on the level splitting Δ.

To finish this section, we evaluate the tunnelling conductance G in the DQWs with separate contacts. This value is defined as $G = \mathrm{d}j/\mathrm{d}V$, where $V = (\varepsilon_{Fl} - \varepsilon_{Fr})/e$ is the applied voltage. We consider two particular cases: (a) ohmic conductance, for which $V \to 0$, $\varepsilon_{Fl} \simeq \varepsilon_{Fr} = \varepsilon_F$, and (b) conductance in symmetric DQWs with equal electron densities [1], for which $eV = \Delta$ and $\varepsilon_{Fl} = \varepsilon_{Fr} + \Delta = \varepsilon_F$. The expressions are as follows:

$$\begin{aligned} G = e^2\rho_{2D}\nu_0 \Bigg\{ 1 + \exp\left(-\frac{\pi}{\omega_c\tau}\right) \Bigg[&\delta^{-1} \left(\sin\left[\frac{2\pi\varepsilon_F}{\hbar\omega_c}\right] - \sin\left[\frac{2\pi(\varepsilon_F + \Delta)}{\hbar\omega_c}\right] \right) \\ &- \frac{2}{1 + \delta^{-2}} \left(\cos\left[\frac{2\pi\varepsilon_F}{\hbar\omega_c}\right] + \cos\left[\frac{2\pi(\varepsilon_F + \Delta)}{\hbar\omega_c}\right] \right) \Bigg] \Bigg\} \end{aligned} \tag{17}$$

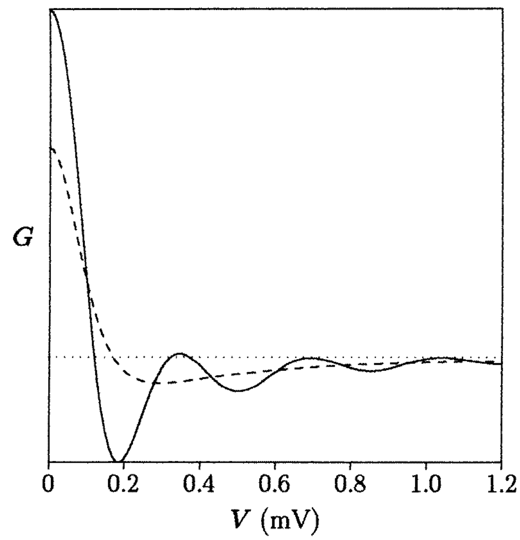

Figure 2. The dependence of the tunnelling conductance G (arbitrary units) on the applied voltage V in separately contacted DQWs with matched electron densities [1] at $H = 0$ (dashed line) and $H = 0.2$ T (solid line).

for case (a), and

$$
\begin{aligned}
G &= G_0 + G_1 \\
G_0 &= e^2 \rho_{2D} \nu_0 (1 - \delta^2)/(1 + \delta^2) \\
G_1 &= e^2 \rho_{2D} \nu_0 \exp\left(-\frac{\pi}{\omega_c \tau}\right) \cos\left(\frac{2\pi\varepsilon_F}{\hbar\omega_c}\right) \left\{ -\frac{2}{\delta} \sin\left(\frac{2\pi\Delta}{\hbar\omega_c}\right) \right. \\
&\quad - \frac{4}{1+\delta^{-2}} \cos\left(\frac{2\pi\Delta}{\hbar\omega_c}\right) + \frac{4\hbar\omega_c}{\pi\Delta} \sin\left(\frac{2\pi\Delta}{\hbar\omega_c}\right) \frac{1-\delta^{-2}}{(1+\delta^{-2})^2} \\
&\quad \left. - \frac{\hbar\omega_c}{\pi\Delta\delta} \left[1 - \cos\left(\frac{2\pi\Delta}{\hbar\omega_c}\right)\right] \frac{3+\delta^{-2}}{1+\delta^{-2}} \right\}
\end{aligned}
\tag{18}
$$

for case (b). In both cases, value of G depends on the Fermi energy ε_F through the factor $2\pi\varepsilon_F/\hbar\omega_c$. Since $\exp[-\pi/\omega_c\tau] \ll 1$, the oscillations of the electron density of states are weak, according to the formula $\rho_{2D}\{1 - 2\exp[-\pi/\omega_c\tau]\cos(2\pi\varepsilon/\hbar\omega_c)\}$ (for the l-well). Therefore, the Fermi energy is connected with the electron concentration in the l-well, n_l, in the same way as in the absence of the magnetic field: $\varepsilon_F = n_l/\rho_{2D}$. Corrections to this relation due to the magnetic field effect are fairly small and do not modify the factor $2\pi\varepsilon_F/\hbar\omega_c$ very much. Figure 2 shows the dependence of the conductance on the applied voltage in case (b), calculated for $H = 0.2$ T, $\hbar/\tau = 0.17$ meV (the conditions of the experiment described in reference [1], figure 3), and the Fermi energy is chosen in order to give the maximum amplitude of the oscillations: $\cos(2\pi\varepsilon_F/\hbar\omega_c) = -1$. A transition from the non-oscillating ($H = 0$, dashed line) to the oscillating ($H \neq 0$, solid line) regime is clear. We note that the behaviour of G versus V is very similar to that observed experimentally in reference [1] (see also reference [11]). The amplitudes of the oscillations shown in figure 3 of reference [1] are higher than those calculated here (this disagreement is possibly because $\exp[-\pi/\omega_c\tau] \ll 1$ is not a very good approximation for the experimental conditions). Nevertheless, the ratio of the amplitudes of the main peak ($V = 0$) at $H = 0$ and $H = 0.2$ T is in good agreement with the experimental data.

4. The case of strong magnetic fields

In this section we consider the case where the Landau levels are well defined, i.e. the Landau level broadening is small in comparison with the cyclotron energy. Since this condition requires rather high magnetic fields (about 10 T for GaAs/AlGaAs structures), we assume that the electrons occupy only the lowest Landau level of the left-hand well. The states of the right-hand well are assumed to be unpopulated. We also neglect spin-dependent effects. This is possible when the spin-splitting energy is smaller than the Landau level widths. For non-degenerate electrons it is enough to assume that the spin-splitting energy is small in comparison with the electron temperature (spin-flip transitions are neglected). The description of the one-particle Green function in a high magnetic field can be based on two different approaches. The first is the path-integral approach, which has been applied in references [12] and [13] for determination of the density of states in high magnetic fields (for an application of the path-integral method to oscillatory-like motion, see also review [14]). The second is the self-consistent Born approximation (SCBA) developed in an application to 2D systems in magnetic fields by Ando (see references in [15] and [10]). Below we calculate the tunnelling relaxation rate and tunnelling current using these approaches and compare the results obtained.

4.1. The path-integral approach

This method is based on an exact expression for the averaged retarded (R) or advanced (A) Green functions $\mathcal{G}_\varepsilon^{jb}(\boldsymbol{x})$ ($b = \mathrm{R, A}$) as

$$\mathcal{G}_\varepsilon^{lR}(\boldsymbol{x}) = -\frac{\mathrm{i}}{\hbar}\int_0^\infty \mathrm{d}t\ \exp\left(\frac{\mathrm{i}}{\hbar}\varepsilon t\right)\int_{\boldsymbol{x}_0=0}^{\boldsymbol{x}_t=\boldsymbol{x}} \mathcal{D}[\boldsymbol{x}_\tau]\ \exp\left\{\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau\ L(\boldsymbol{x}_\tau, \dot{\boldsymbol{x}}_\tau) - \frac{1}{2\hbar^2}\int_0^t\int_0^t \mathrm{d}\tau\ \mathrm{d}\tau'\ W_l(|\boldsymbol{x}_\tau - \boldsymbol{x}_{\tau'}|)\right\} \tag{19}$$

where $\mathcal{D}[\boldsymbol{x}_\tau]$ implies integration over all paths coming from the point $\boldsymbol{x}_0 = 0$ to $\boldsymbol{x}_t = \boldsymbol{x}$, the random potential correlators appearing after averaging over all realizations of the random potentials $U_j(\boldsymbol{x})$ are defined as $W_j(|\boldsymbol{x}|) = \langle U_j(\boldsymbol{x})U_j(0)\rangle$, and

$$L(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \frac{m}{2}\dot{\boldsymbol{x}}^2 + \frac{e}{2c}\boldsymbol{H}\cdot[\boldsymbol{x}\times\dot{\boldsymbol{x}}] \tag{20}$$

is the Lagrangian describing free motion in the magnetic field. The expression for $\mathcal{G}_\varepsilon^{rR}(\boldsymbol{x})$ differs from equation (19) by a change of the well index ($l \to r$) and an energy shift $\varepsilon \to \varepsilon + \Delta$. The advanced Green functions are written in analogy with the retarded ones. Further evaluation of equation (19) may be done only under some approximations. In the following, we assume that the characteristic scales of the disorder potential (correlation lengths l_l and l_r) are large in comparison with the Landau orbit radius:

$$l_l, l_r \gg l_H. \tag{21}$$

In the lowest order of the disorder smoothness, the potential correlators $W_j(|\boldsymbol{x}|)$ in the exponent under the path integral are just replaced by the constants $W_j = W_j(0)$, and the causal Green function $G_\varepsilon^l(n)$ is given by

$$G_\varepsilon^l(n) = \frac{1}{\sqrt{2\pi W_l}}\exp\left[-\frac{(\varepsilon - \varepsilon_n)^2}{2W_l}\right] \tag{22}$$

($G^r_\varepsilon(n)$ may be written in a similar way). However, to describe tunnelling between the Landau levels with different numbers, we should search for $G^r_\varepsilon(n)$ in a more elaborate way. To do this, we expand the potential part of the exponential term under the path integral for $\mathcal{G}^{rR}_\varepsilon(\boldsymbol{x})$ in a series for

$$\left[\int_0^t \mathrm{d}\tau \int_0^t \mathrm{d}\tau' \ W_r(|\boldsymbol{x}_\tau - \boldsymbol{x}_{\tau'}|) - W_r t^2\right] \Big/ 2\hbar^2$$

up to the first order. After this transformation, the path integral can be exactly calculated in a way similar to the one described in reference [16] for the 3D systems at $H = 0$. Substituting in Gaussian correlators $W_j(x) = W_j \exp(-x^2/l_j^2)$, we calculate the integrals over coordinate $\boldsymbol{x}$ and time t (the latter is calculated under the approximation $\sqrt{W_r} \ll \hbar\omega_c$) and obtain

$$\int \mathrm{d}\boldsymbol{x} \ \exp\left(-\frac{x^2}{4l_H^2}\right) G^r_\varepsilon(\boldsymbol{x}) \simeq \frac{1}{\sqrt{2\pi W_r}} \left\{ \exp\left[-\frac{(\varepsilon + \Delta - \hbar\omega_c/2)^2}{2W_r}\right] + \frac{W_r}{(\hbar\omega_c)^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left(2\frac{l_H^2}{l_r^2}\right)^n \exp\left[-\frac{(\varepsilon + \Delta - \hbar\omega_c(n + 1/2))^2}{2W_r}\right] \right\}. \tag{23}$$

In this equation we have retained small values of order $2l_H^2/l_r^2$ in the second term of the right-hand side. This term is responsible for the coupling between the lowest Landau level of the l-well (it is situated at $\varepsilon \simeq \hbar\omega_c/2$) and the nth ($n \geqslant 1$) Landau level of the r-well, occurring under resonant magnetotunnelling conditions (1). Coupling between the lowest Landau levels of the l- and r-wells is described by the first term. However, this kind of coupling is unimportant when the splitting energy Δ considerably exceeds the Landau level broadening energy, and is not studied in this section. With the use of equation (23) we can calculate the tunnelling rate (see [8]):

$$\nu = \nu_1 \left(\frac{\Delta}{\hbar\omega_c}\right)^2 \sum_{n=1}^{\infty} \exp\left[-\frac{(\Delta - \hbar\omega_c n - W_l/T_e)^2}{2(W_l + W_r)}\right] \frac{1}{n^2} \left(2\frac{l_H^2}{l_r^2}\right)^n \qquad \nu_1 = \frac{\sqrt{2\pi} W_r T^2}{\hbar\sqrt{W_l + W_r}\,\Delta^2} \tag{24}$$

where we have neglected the exponentially small contribution arising from the first term in the right-hand part of (23). The remaining sum describes the series of the symmetrical peaks caused by the resonant magnetotunnelling. The term W_l/T_e in the exponent describes the temperature-induced shift of the peaks, and can be neglected when the temperature is higher than the Landau level broadening (the case of uniform occupation).

In a similar way, calculation of the tunnelling current for low temperatures gives

$$j = j_1 \sum_{n=1}^{\infty} \frac{1}{2} \left\{ 1 + \mathrm{erf}\left[\sqrt{\frac{W_l + W_r}{2W_l W_r}} \left(\varepsilon_F + \frac{(\Delta - \hbar\omega_c n) W_l}{W_l + W_r}\right)\right]\right\} \times \frac{1}{n^2} \left(2\frac{l_H^2}{l_r^2}\right)^n \exp\left[-\frac{(\Delta - \hbar\omega_c n)^2}{2(W_l + W_r)}\right] \qquad j_1 = \frac{e}{\pi l_H^2} \nu_1 \left(\frac{\Delta}{\hbar\omega_c}\right)^2 \tag{25}$$

where erf is the error function, and ε_F is connected with the electron concentration of the l-well according to equation

$$n_l = \frac{1}{2\pi l_H^2} \left[1 + \mathrm{erf}\left(\frac{\varepsilon_F}{\sqrt{2W_l}}\right)\right] \tag{26}$$

describing the dependence of the lowest l-well Landau level occupation on the magnetic field. When this level is fully occupied, $\mathrm{erf}\cdots$ in equation (25) should be replaced by 1, and we have $j = en_l\nu$, where ν is given by equation (24) with $T_e \to \infty$.

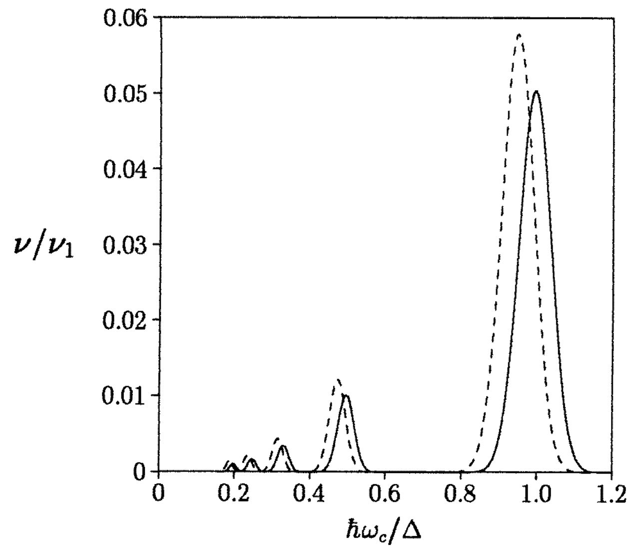

Figure 3. The dependence of the tunnelling rate on the magnetic field in the case of long-range scattering potentials (obtained by the path-integral method) at two different temperatures: $T_e \to \infty$ (solid line) and $\sqrt{W}/T_e = 1.5$ (dashed line). The splitting energy Δ is fixed.

Figure 3 illustrates the dependence of ν/ν_1 (see equation (24)) on the cyclotron energy $\hbar\omega_c$ when Δ is a constant. The calculation is done at $W_r = W_l = W$ with the use of the following dimensionless parameters: $2\hbar^2/(ml_r^2\Delta) = 0.05$, $\sqrt{W}/\Delta = 0.03$, $T_e \to \infty$ (solid line) and $\sqrt{W}/T_e = 1.5$ (dashed line). Several peaks shown here correspond to the resonances described by equation (1) with $n = 0$, $n' = 1, 2, 3, 4, 5$. The amplitudes of the peaks decrease rather quickly with the increasing n', because the transitions between the Landau levels in conditions (21) are rapidly suppressed when $n' - n$ increases. The relaxation rate ν also shows peaks as a function of the splitting energy Δ.

In figure 4 we show dependence of j/j_1 (see equation (25)) on the level-splitting energy Δ in conditions in which the lowest l-well Landau level is half-filled (i.e. ε_F in equation (25) is equal to 0). The calculation is done at $W_r = W_l = W$ with $l_H^2/l_r^2 = 0.3$, and $\sqrt{W}/(\hbar\omega_c) = 0.1$. Due to non-uniform occupation of the l-well Landau level (this is reflected by the term $\{1 + \mathrm{erf}\cdots\}$ of equation (25)) the peak positions are shifted from $\Delta = n\hbar\omega_c$ to higher Δ. Similar peaks can be found in the magnetic field dependence of the tunnelling current.

4.2. The self-consistent Born approximation

The SCBA implies solution of the Dyson diagrammatic equation, which is written in the Landau level representation as

$$\mathcal{G}_\varepsilon^{jR,A}(n) = \mathcal{G}_\varepsilon^{j(0)}(n) + \mathcal{G}_\varepsilon^{j(0)}(n)\mathcal{G}_\varepsilon^{jR,A}(n)\sum_{n'}\Phi_{nn'}^{j}\mathcal{G}_\varepsilon^{jR,A}(n') \tag{27}$$

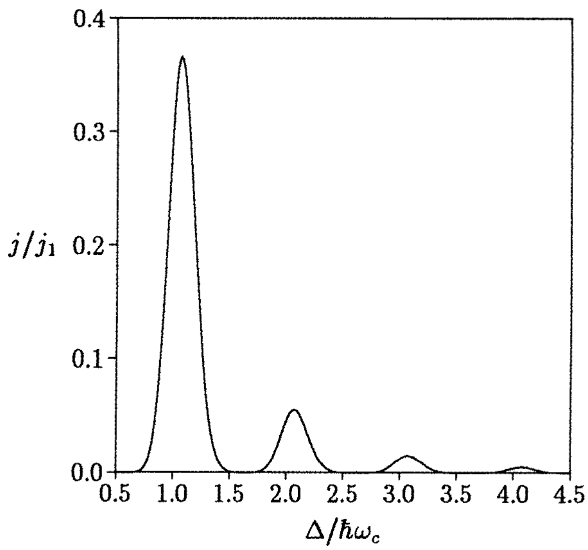

Figure 4. The dependence of the tunnelling current on the splitting energy Δ at a fixed magnetic field corresponding to half-filling of the lowest Landau level in the left-hand well (obtained by the path-integral method).

where $\mathcal{G}^{l(0)}_{\varepsilon}(n) = (\varepsilon - \varepsilon_n)^{-1}$ and $\mathcal{G}^{r(0)}_{\varepsilon}(n) = (\varepsilon + \Delta - \varepsilon_n)^{-1}$ are the Green functions of free motion,

$$\Phi^{j}_{nn'} = \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^2}\ |Q_{nn'}(q)|^2 W_j(q) \tag{28}$$

$$|Q_{nn'}(q)|^2 = \frac{n!}{n'!} \exp[-(ql_H)^2/2] \left[\frac{(ql_H)^2}{2}\right]^{n'-n} \left[L_n^{n'-n}\left(\frac{(ql_H)^2}{2}\right)\right]^2 \tag{29}$$

and $W_j(q)$ are the Fourier transforms of the scattering potential correlators $W_j(|\boldsymbol{x}|)$. The Dyson equation is obtained as a result of a partial summation of the diagrams, and it is not exact. However, it is rather easy to solve this equation in the case of high H considered here. If we do not take into account intermixing between the different Landau levels, we obtain $G^{j}_{\varepsilon}(n)$ as a 'semi-elliptical' peak [15] centred near the position of the nth Landau level. In order to describe the tunnelling between the Landau levels with different numbers, we should calculate $G^{j}_{\varepsilon}(n)$ taking into account all terms in the sum over n' in equation (27). Such a calculation is done by iterations on the small parameters of order $\Phi^{j}_{nn'}/\hbar\omega_c$. The result is given by (here and below $\Phi^{j}_{n} \equiv \Phi^{j}_{nn}$)

$$\begin{aligned} G^{l}_{\varepsilon}(n) = \frac{1}{\pi}\theta\left(2\sqrt{\Phi^{l}_{n}} - |\varepsilon_n - \varepsilon|\right) \\ \times \left\{\sqrt{\frac{1}{\Phi^{l}_{n}} - \frac{(\varepsilon - \varepsilon_n)^2}{4\Phi^{l2}_{n}}} + \frac{1}{2\Phi^{l}_{n}} \frac{\varepsilon_n - \varepsilon}{\sqrt{4\Phi^{l}_{n} - (\varepsilon_n - \varepsilon)^2}} \sum_{n'} \frac{\Phi^{l}_{nn'}}{\varepsilon_{n'} - \varepsilon_n}\right\} \\ + \frac{1}{\pi} \sum_{n' \neq n} \frac{\Phi^{l}_{nn'}}{(\varepsilon_n - \varepsilon_{n'})^2} \sqrt{\frac{1}{\Phi^{l}_{n'}} - \frac{(\varepsilon - \varepsilon_{n'})^2}{4\Phi^{l2}_{n'}}}\,\theta\left(2\sqrt{\Phi^{l}_{n'}} - |\varepsilon_{n'} - \varepsilon|\right) \end{aligned} \tag{30}$$

($G^r_\varepsilon(n)$ is described in an analogous way), and we see that a number of smaller semi-elliptical peaks arise in addition to the main peak. The tunnelling rate is expressed as

$$\nu = \frac{2T^2}{\hbar^3\omega_c^2}\sum_{n=1}^{\infty}\frac{\Phi^r_{0n}}{\sqrt{\Phi^r_n}n^2}\int \mathrm{d}\varepsilon\ \exp\left(-\frac{\varepsilon}{T_e}\right)\sqrt{1-\frac{\varepsilon^2}{4\Phi^l_0}}\sqrt{1-\frac{(\varepsilon+\Delta-n\hbar\omega_c)^2}{4\Phi^r_n}}$$
$$\times\left[\int \mathrm{d}\varepsilon\ \exp\left(-\frac{\varepsilon}{T_e}\right)\sqrt{1-\frac{\varepsilon^2}{4\Phi^l_0}}\right]^{-1}. \qquad (31)$$

The limits of integration in this equation are determined by the requirement that the expressions under the square roots must be positive. The tunnelling current j is given by

$$j = \frac{2eT^2}{\pi^2\hbar^3 l_H^2\omega_c^2}\sum_{n=1}^{\infty}\frac{\Phi^r_{0n}}{\sqrt{\Phi^r_n\Phi^l_0}n^2}\int^{\varepsilon_F}\mathrm{d}\varepsilon\ \sqrt{1-\frac{\varepsilon^2}{4\Phi^l_0}}\sqrt{1-\frac{(\varepsilon+\Delta-n\hbar\omega_c)^2}{4\Phi^r_n}} \qquad (32)$$

where ε_F is obtained according to

$$n_l = \frac{1}{2\pi l_H^2}\left[\frac{\varepsilon_F}{\pi\sqrt{\Phi^l_0}}\sqrt{1-\frac{\varepsilon_F^2}{4\Phi^l_0}}+\left(1+\frac{2}{\pi}\sin^{-1}\frac{\varepsilon_F}{2\sqrt{\Phi^l_0}}\right)\right]. \qquad (33)$$

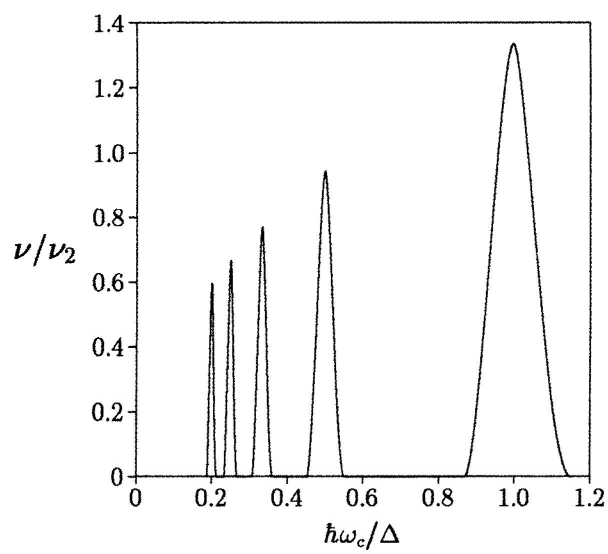

Figure 5. The dependence of the tunnelling rate on the magnetic field in the case of short-range scattering potentials (obtained using the SCBA) at $T_e \to \infty$. The splitting energy Δ is fixed.

In the same way as equations (24) and (25), equations (31) and (32) show peaks in the resonant magnetotunnelling conditions (1). The magnetic field dependence of ν from equation (31) is presented in figure 5 (the five highest peaks are shown here). In the calculation we have taken $T_e \to \infty$ and applied the approximation of a short-range scattering potential symmetrically distributed across the DQWs, which gives $W_j(q) = w$ and

$\Phi^j_{nn'} = \Phi = w/(2\pi l_H^2)$. The tunnelling rate is expressed in units of ν_2, where

$$\nu_2 = \frac{2T^2}{\pi\hbar\Delta}\sqrt{\frac{2wm}{\pi\hbar^2\Delta}} \tag{34}$$

and we have chosen $\sqrt{2wm/(\pi\hbar^2\Delta)} = 0.07$ for the numerical calculation. In contrast to the results of the path-integral method (see figure 3), the peaks are not Gaussian. The peak heights decrease with the increase of the r-well Landau level number more slowly than in figure 3, because we have used here the short-range potential approximation, in contrast to the long-range potential approximation described by conditions (21). In order to compare the results of the SCBA and the path-integral methods, we have calculated the peak heights from equation (31) for conditions (21), for which $\Phi^j_n = W_j$ and $\Phi^r_{0n} = (2l_H^2/l_r^2)^n W_r$. We find the same n-dependence of the peak heights (determined by the factor $(1/n^2)(l_H/l_r)^{2n}$), while the absolute values of the peak heights differ from the result of the path-integral method (see equation (24)) by the numerical factor $16/(3\pi^{3/2}) \simeq 0.96$ which is close to 1. From this we conclude that the SCBA may be applied for calculation of the peak heights very well. On the other hand, the SCBA is not good in the description of the edges of the density of states, and the magnetotunnelling peaks, following the cut-offs of the density of states, show non-physically sharp edges.

5. The influence of the interwell correlation of the scattering

In previous section we have calculated the magnetotunnelling of electrons from the lowest Landau level of the l-well to a set of Landau levels of r-well. Although the tunnelling between the lowest Landau levels of the l- and r-wells has not been studied explicitly, the above consideration shows that calculation of ν and j in this situation is reduced simply to integration of the product of the densities of states in the l- and r-wells over the energy. Since the approximations of the densities of states given by the path-integral formalism in conditions (21) (Gaussian peaks) and SCBA (semi-elliptical peaks) are well known [15], this problem appears to be simple. It becomes less trivial if the interwell correlation of the scattering potential is taken into account. Such a calculation, which demonstrates the influence of this correlation, is presented below.

5.1. The path-integral method

Application of the path-integral method to calculation of the pair correlators from equations (2) and (3) implies expression of non-averaged Green functions $G^j_\varepsilon(\boldsymbol{x}', \boldsymbol{x})$ through the path integrals (see [14] or [16]) and subsequent statistical averaging [16] as described below:

$$\begin{aligned}\left\langle \exp\left\{-\frac{\mathrm{i}}{\hbar}\int_0^t \mathrm{d}\tau\ U_l(\boldsymbol{x}_\tau) - \frac{\mathrm{i}}{\hbar}\int_0^{t'} \mathrm{d}\tau\ U_r(\boldsymbol{x}_\tau)\right\}\right\rangle \\ = \exp\left\{-\frac{1}{2\hbar^2}\int_0^t\int_0^t \mathrm{d}\tau\ \mathrm{d}\tau'\ W_l(|\boldsymbol{x}_\tau - \boldsymbol{x}_{\tau'}|)\right. \\ - \frac{1}{2\hbar^2}\int_0^{t'}\int_0^{t'} \mathrm{d}\tau\ \mathrm{d}\tau'\ W_r(|\boldsymbol{x}_\tau - \boldsymbol{x}_{\tau'}|) \\ \left. - \frac{1}{\hbar^2}\int_0^t\int_0^{t'} \mathrm{d}\tau\ \mathrm{d}\tau'\ W_{lr}(|\boldsymbol{x}_\tau - \boldsymbol{x}_{\tau'}|)\right\}\end{aligned} \tag{35}$$

where the interwell potential correlator $W_{lr}(|\boldsymbol{x}|) = \langle U_l(\boldsymbol{x})U_r(0)\rangle$ has appeared in addition to the intrawell correlators $W_l(|\boldsymbol{x}|)$ and $W_r(|\boldsymbol{x}|)$. In conditions (21), we put $W_j(|\boldsymbol{x}|) = W_j$ and $W_{lr}(|\boldsymbol{x}|) = W_{lr}$ and obtain

$$\frac{1}{S}\int\int \mathrm{d}\boldsymbol{x}\ \mathrm{d}\boldsymbol{x}'\ \left\langle G^l_\varepsilon(\boldsymbol{x},\boldsymbol{x}')G^r_\varepsilon(\boldsymbol{x}',\boldsymbol{x})\right\rangle = -\frac{1}{(8\pi^2\hbar l_H^2)^2}\int \mathrm{d}t\int \mathrm{d}t'\ \exp\left[\frac{\mathrm{i}}{\hbar}\varepsilon t + \frac{\mathrm{i}}{\hbar}(\varepsilon+\Delta)t'\right]$$
$$\times\ \frac{\exp(-W_l t^2/2\hbar^2 - W_r t'^2/2\hbar^2 - W_{lr}tt'/\hbar^2)}{\sin(\omega_c t/2)\sin(\omega_c t'/2)}$$
$$\times \int \mathrm{d}\boldsymbol{x}\ \exp\left[\frac{\mathrm{i}\boldsymbol{x}^2}{4l_H^2}\left(\cot\frac{\omega_c t}{2} + \cot\frac{\omega_c t'}{2}\right)\right]. \tag{36}$$

Calculation of the integrals over $\boldsymbol{x}$, t and t' in equation (36) and integrals over ε in (2) and (3) gives

$$\nu = \frac{\sqrt{2\pi}T^2}{\hbar\sqrt{W_l+W_r-2W_{lr}}}\exp\left[-\frac{(\Delta-(W_l-W_{lr})/T_e)^2}{2(W_l+W_r-2W_{lr})}\right] \tag{37}$$

and

$$j = \frac{\sqrt{2}eT^2}{\sqrt{\pi(W_l+W_r-2W_{lr})}\hbar l_H^2}\exp\left[-\frac{\Delta^2}{2(W_l+W_r-2W_{lr})}\right]$$
$$\times\ \frac{1}{2}\left\{1+\mathrm{erf}\left[\sqrt{\frac{W_l+W_r-2W_{lr}}{2(W_lW_r-W_{lr}^2)}}\left(\varepsilon_F+\frac{\Delta(W_l-W_{lr})}{W_l+W_r-2W_{lr}}\right)\right]\right\} \tag{38}$$

where ε_F is connected with n_l according to (26). Expression (37) describes the dependence of the tunnelling relaxation rate on the splitting energy Δ (a Gaussian peak). It also shows that the interwell correlation of the scattering potential tends to increase the peak height and to make the peak less broad. The tunnelling current (38) has a similar behaviour.

5.2. *The self-consistent Born approximation*

Below we make use of the following expression:

$$\frac{1}{S}\int\int \mathrm{d}\boldsymbol{x}\ \mathrm{d}\boldsymbol{x}'\ \left\langle G^l_\varepsilon(\boldsymbol{x},\boldsymbol{x}')G^r_\varepsilon(\boldsymbol{x}',\boldsymbol{x})\right\rangle = \frac{1}{(2\pi l_H)^2}\sum_n[\Pi^{RA}_n + \Pi^{AR}_n - \Pi^{AA}_n - \Pi^{RR}_n] \tag{39}$$

where $\Pi^{bb'}_n$ are the pair correlators of the retarded and advanced Green functions in the Landau level representation. Then we search for $\Pi^{bb'}_n$ in the ladder approximation, which gives rise to the equation

$$\Pi^{bb'}_n = \mathcal{G}^{lb}_\varepsilon(n)\mathcal{G}^{rb'}_\varepsilon(n)\left[1+\sum_{n'}\Phi^{lr}_{nn'}\Pi^{bb'}_{n'}\right] \tag{40}$$

where $\Phi^{lr}_{nn'}$ is given by equation (28) where $W_j(q)$ is replaced by $W_{lr}(q)$. Neglecting intermixing between the different Landau levels in equation (40), we put $n' = n$ in the sum. Further calculations are straightforward. The tunnelling relaxation rate and tunnelling current are given by

$$\nu = \frac{2T^2}{\hbar\sqrt{\Phi^r_0}}\int \mathrm{d}\varepsilon\ \exp\left(-\frac{\varepsilon}{T_e}\right)K(\varepsilon)\sqrt{1-\frac{\varepsilon^2}{4\Phi^l_0}}\sqrt{1-\frac{(\varepsilon+\Delta)^2}{4\Phi^r_0}}$$
$$\times\ \left[\int \mathrm{d}\varepsilon\ \exp\left(-\frac{\varepsilon}{T_e}\right)\sqrt{1-\frac{\varepsilon^2}{4\Phi^l_0}}\right]^{-1} \tag{41}$$

$$j = \frac{2eT^2}{\pi^2\hbar l_H^2\sqrt{\Phi_0^l\Phi_0^r}}\int^{\varepsilon_F} \mathrm{d}\varepsilon\ K(\varepsilon)\sqrt{1-\frac{\varepsilon^2}{4\Phi_0^l}}\sqrt{1-\frac{(\varepsilon+\Delta)^2}{4\Phi_0^r}}. \tag{42}$$

Here ε_F, as before, is connected with n_l by equation (33), and the function of interwell correlation $K(\varepsilon)$ is given by

$$K(\varepsilon) = \frac{1-\gamma^2}{(1-\gamma^2)^2 - \gamma(1+\gamma^2)\varepsilon(\varepsilon+\Delta)\Big/\sqrt{\Phi_0^l\Phi_0^r} + \gamma^2[\varepsilon^2/\Phi_0^l + (\varepsilon+\Delta)^2/\Phi_0^r]} \tag{43}$$

where $\gamma = \Phi_0^{lr}\Big/\sqrt{\Phi_0^l\Phi_0^r}$. If $\Phi_{nn}^{lr} = 0$, $K(\varepsilon) = 1$. The function $K(\varepsilon)$ at $\Delta = 0$ and $\Phi_0^l = \Phi_0^r$ has been previously calculated in reference [17] in an application to the problem of magnetoconductivity of the multiple-quantum-well system.

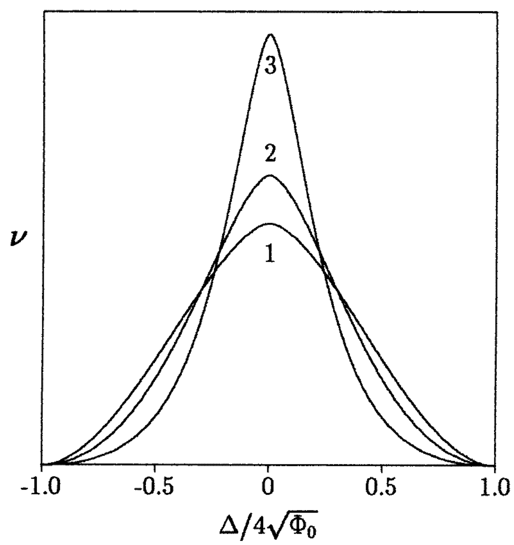

Figure 6. The dependence of the tunnelling rate ν (arbitrary units) on Δ in the case of tunnelling between the lowest Landau levels (obtained using the SCBA). The narrowing of the peak with the increase of the interwell scattering potential correlation ($\gamma = 0$ (curve 1), $\gamma = 0.2$ (curve 2), and $\gamma = 0.5$ (curve 3)) is illustrated.

The influence of the interwell correlation described by equations (41) and (42) is the same as was found in the previous subsection: the peak dependence of ν and j on Δ becomes higher and narrower. This behaviour is illustrated in figure 6, where we plot the dependence of ν on Δ for several values of the interwell correlation parameter γ under symmetric scattering ($\Phi_0^l = \Phi_0^r = \Phi_0$) and high-temperature ($T_e \to \infty$) conditions. Comparison of the SCBA value of the relaxation rate peak height for conditions (21) with the proper value obtained by the path-integral method shows that the ratio of these values is close to 1 up to $\gamma = 0.5$.

6. Conclusions

In this paper we have presented an analytical calculation of the elastic scattering-assisted magnetotunnelling between the weakly coupled 2D gases, and demonstrated oscillating behaviour of the tunnelling relaxation rate ν and tunnelling current j as functions of the magnetic field H, splitting energy Δ, and (in the case of low H) Fermi energies of the electrons in the wells. At low H, for which the Landau quantization is suppressed by the scattering, the oscillating part of the current appears as a small correction to the background current ($H = 0$) and can be considered by the methods usually used in the description of the other magneto-oscillatory phenomena (for example, Shubnikov–de Haas oscillations). In strong H, for which the Landau levels are well defined, the current and the relaxation rate show sharp peaks appearing in resonant magnetotunnelling conditions (1). Both the low-field oscillations and the resonant magnetotunnelling peaks of the current have been observed experimentally (see the references cited above) and their properties are in agreement with the results of calculations presented here.

If Δ considerably exceeds the Landau level broadening and the magnetic field is strong, the electrons may tunnel only due to transitions between the Landau levels with different numbers ($n \neq n'$). These transitions are possible due to the scattering. As a result, the peak heights are small according to a small parameter: the ratio of the Landau level broadening energy to the cyclotron energy. The peak heights decrease with the increase of $n - n'$; the rate of this decrease is 'faster' for the long-range-correlated scattering potentials. These properties are reflected by equations (24), (25), (31) and (32). Although there have been discussions concerning scattering-assisted tunnelling between the Landau levels (see, for example, [6] and [5]), a detailed theoretical investigation of these processes, as far as we know, has not been presented.

The mixing of the Landau levels with different numbers is possible without scattering, if, apart from the perpendicular field H, there is also a magnetic field $H_{\parallel}$ applied parallel to the layers. If this field is not very strong, and so the cyclotron energy $\hbar e H_{\parallel}/mc$ is small in comparison with the size-quantization energies of the electrons in the wells, the eigenstates of the system are not modified significantly (the single-well eigenstates in a strong parallel magnetic field have been studied in reference [18]) and the general approach described in section 2 remains valid. In the presence of $H_{\parallel}$, the right-hand side of equation (7) is modified as

$$\frac{1}{2\pi l_H^2} \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} \frac{n!}{n'!} \mathrm{e}^{-\beta} \beta^{(n'-n)} \left[L_n^{n'-n}(\beta) \right]^2 G_\varepsilon^l(n) G_\varepsilon^r(n') \tag{44}$$

where $\beta = H_{\parallel}^2 Z^2/(2H^2 l_H^2)$ is the dimensionless parameter associated with the in-plane magnetic field and Z is the distance between the centres of the wave functions in the wells. When β is small, the coupling between the Landau levels with numbers 0 and n is determined by the factor $\beta^n/n!$. When, with the increase of $H_{\parallel}$, this factor becomes comparable with the ratio $\Phi_{0n}^r/(n\hbar\omega_c)^2$ (the single-well eigenstates are still not modified in this field), the influence of the in-plane magnetic field on the tunnelling between the 0th and nth Landau levels must be taken into account, and the magneto-oscillation peak heights should be changed substantially.

If Δ is smaller than the Landau level broadening (such a small Δ corresponds to the resonant tunnelling conditions at $H = 0$), the tunnelling occurs between the Landau levels with equal numbers (if only the lowest Landau level is occupied, $n = n' = 0$). The probability of this tunnelling is high, and the maximum tunnelling relaxation rate is inversely proportional to the Landau level broadening energy (see, for example, equation (37)). We have studied the influence of the interwell potential correlations on the dependence on Δ of ν and j in this situation (i.e. on the shape of the main magnetotunnelling peak) and have found that it leads to narrowing of the peak, like it does with the resonant tunnelling peak at $H = 0$ [19]. The same effect should be expected for tunnelling between the Landau levels with different numbers.

Let us discuss the approximations used above. The main approximation is the one-particle approach to the problem on the assumption that the Coulomb interaction of the electrons is not essential for determination of the tunnelling current. This statement is not true for clean enough systems at low temperatures, where the Coulomb interaction has a dramatic influence on the electron spectrum. The many-electron phenomena are manifested in the fractional quantum Hall effect [20]. They are also responsible for the arising of such ordered (correlated) states as the Wigner lattice and charge-density wave, which are currently under investigation in low-dimensional systems. Experimental data [4] on the low-temperature tunnelling between high-mobility 2D electron layers show some shift of the main magnetotunnelling peak, which reflects a gap for tunnelling. This phenomenon has been explained [21, 22] in terms of Wigner lattice formation in high magnetic fields. The effect described lies beyond the one-particle approach developed in this paper. However, the one-particle approach can be satisfactorily applied for the description of the magnetotunnelling in 'dirty' systems, where the scattering is so strong that it suppresses any Coulomb-induced ordering of the electrons (in other words, the scattering-induced broadening of the Landau levels must be larger than the gaps arising due to the ordering).

In the description of DQWs with separate contacts we have used the approximation of a homogeneous distribution of the tunnelling current in the DQWs plane. Investigation [23] of inhomogeneous current distributions at $H = 0$ provides proper estimates for the validity of this approximation, which can be applied for the quasiclassical (low-magnetic-field) case of section 3. However, the question about the modification of these estimates in quantizing magnetic fields remains open.

In conclusion, our calculations demonstrate possibilities of application of the tunnelling magneto-oscillations for characterization of the scattering potentials in the wells and examination of the electron density of states in strong magnetic fields. The density of states in the magnetic field has been extensively studied with the use of the above-described approximation for the one-particle Green functions; see [12], [13] and references in [15]. In this paper we have applied these approximations for calculation of the tunnelling relaxation rate and tunnelling current. If the interwell correlation of the scattering potential is neglected, this application is rather straightforward, although it requires a more careful evaluation of the Green functions, including the corrections due to scattering-induced 'mixing' of different Landau levels. We have explained how these corrections should be taken into account both in the SCBA and in the path-integral approach, and discussed the shape of the magnetotunnelling peaks in these approximations (section 4), describing its connection to the shape of the density of states. The problem of the calculation of the tunnelling magneto-oscillations is more complex in the presence of interwell correlation, for which two-particle correlators of the Green functions should be evaluated (section 5). In this case, the magnetotunnelling peaks appear to be narrower and their connection to the density of states is less direct. A comparison between the experimentally determined widths of the magnetotunnelling peaks and densities of states can provide information about the interwell correlation of the scattering potential in DQWs.

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